25,000). As G increases, the period of the oscillations decreases from 0.89 to 0.24 as G varies from 10,030 to 20,000.

## NOTATION

x and y, Cartesian coordinates; $\psi$, stream function; $\varphi$, velocity vortex; T, temperature; $\eta$, coefficient of dynamic viscosity; $\nu$, coefficient of kinematic viscosity; $\varkappa$, coefficient of thermal conductivity; $\beta$, coefficient of volume expansion; $\theta$, temperature difference; G, Grashof's number; P, Prandtl's number; L and $a$, geometric parameters; $\chi$, coefficient of thermal diffusivity. The indices 1 and 2 refer to the upper and lower liquids, respectively.

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# APPLICATION OF THE PROJECTION-NET METHOD FOR SOLVING THE TRANSIENT HEAT-TRANSFER PROBLEM IN AN ANNULAR DUCT OF COMPLEX CONFIGURATION

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The influence of the geometrical characteristics of ducts on various parameters of the heat-transfer processes taking place in them is investigated.

A topic of practical importance in the study of heat conduction and convective heat transfer is the influence of the geometrical characteristics of the investigated objects, ducts in particular, on various parameters of the processes involved [1]. To study the dependence of the temperature field on the geometrical characteristics and to obtain a realistic picture of the heat-transfer processes in a duct it is necessary to investigate simultaneously the processes of heat conduction in the wall and heat transfer in the fluid, i.e., to solve the problem in the conjugate formulation [2].

Analytical methods for the solution of conjugate transient (time-dependent) convective heat-transfer problems have not been adequately developed [2, 3], and their application is rendered difficult by the need to allow for the cross-sectional geometry and the boundary conditions specified on the outer surface of the wall. In our opinion, therefore, the projection-net method is the most promising approach to the solution of the indicated problems; it combines the finite-element method (FEM) with the finite-difference method (FDM).

The inherent capability of using irregular nets in the FEM permits the curvilinear boundaries of the computational domain to be effectively approximated, and the variational formulation of the problem makes it easy to take various types of boundary conditions into account. Another advantage of the FEM is the feasibility of forming the system of equations automatically; this is achieved by inspecting each element separately and applying a conditioning procedure that will ensure continuity of the function at the interelement boundaries. The FDM ensures the necessary speed and accuracy of the computations in analyzing the behavior of the heat transfer with time and in the direction of motion of the fluid.

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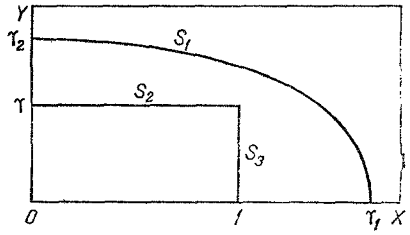


Fig. 1. Cross-sectional domain of the duct:
$S_1: \left\{0 \leqslant X \leqslant \gamma_1;\ \frac{X^2}{\gamma_1^2} + \frac{Y^2}{\gamma_2^2} = 1\right\}$;
$S_2: \{0 \leqslant X \leqslant 1;\ Y = \gamma\}$; $S_3: \{X = 1;\ 0 \leqslant Y \leqslant \gamma\}$.

We consider hydrodynamically stabilized laminar fluid flow in an annular duct having a complex cross section (Fig. 1). We assume that the temperature field in the fluid flow is uniform at the initial time, the wall temperature at the entry is equal to the fluid temperature $t_0$ at the initial time, the duct is thermally insulated at the exit, heat sources are absent in the duct, energy dissipation is negligible, and the thermophysical properties of the fluid and the material of the duct walls are constant.

Then the transient heat-transfer process in the duct with allowance for heat conduction in the wall is described mathematically in Cartesian coordinates by the equations

$$\frac{\partial \Theta_1}{\partial \mathrm{Fo}} = \frac{\partial^2 \Theta_1}{\partial X^2} + \frac{\partial^2 \Theta_1}{\partial Y^2} + \frac{\partial^2 \Theta_1}{\partial Z^2} \tag{1}$$

$$\left(\mathrm{Fo} > 0;\ Z > 0;\ 1 \leqslant X \leqslant \gamma_1;\ \frac{X^2}{\gamma_1^2} + \frac{Y^2}{\gamma_2^2} = 1\right),$$

$$\Theta_1(X,\ Y,\ 0,\ \mathrm{Fo}) = \Theta_1(X,\ Y,\ Z,\ 0) = 0; \tag{2}$$

$$\frac{1}{k_a}\frac{\partial \Theta_2}{\partial \mathrm{Fo}} + W_Z \mathrm{Pe}\frac{\partial \Theta_2}{\partial Z} = \frac{\partial^2 \Theta_2}{\partial X^2} + \frac{\partial^2 \Theta_2}{\partial Y^2} + \frac{\partial^2 \Theta_2}{\partial Z^2} \tag{3}$$

$$(\mathrm{Fo} > 0;\ Z > 0;\ 0 \leqslant X \leqslant 1;\ 0 \leqslant Y \leqslant \gamma),$$

$$\Theta_2(X,\ Y,\ 0,\ \mathrm{Fo}) = \Theta_2(X,\ Y,\ Z,\ 0) = 0; \tag{4}$$

$$\left.\frac{\partial \Theta_1(X,\ Y,\ Z,\ \mathrm{Fo})}{\partial n}\right|_{Z\to\infty} = \left.\frac{\partial \Theta_2(X,\ Y,\ Z\ \ \mathrm{Fo})}{\partial n}\right|_{Z\to\infty} = 0,$$

where $X = x/a$, $Y = y/a$, $Z = z/a$, $\mathrm{Fo} = a_1\tau/a^2$, $k_a = a_2/a_1$, $\gamma = b/a$, $\mathrm{Pe} = \bar{w}a/a_2$. Dirichlet, Neumann, or Cauchy boundary conditions can be specified on the outer surface of the duct wall ($S_1$). Conjugation conditions ("boundary conditions of the fourth kind") are specified at the boundary between the fluid and the wall $(S_2 \cup S_3)$.

The problem (1), (3) with the uniqueness conditions (initial conditions, boundary conditions, and conditions at the entry and exit) is solved numerically by the projection-net method. We partition the cross-sectional domain of the duct into triangular simplex elements [4], the application of which specifies a linear approximation of the temperature

$$T^{(e)} = [N^{(e)}]\{T\} \tag{5}$$

on each finite element [5].

Using the procedure of the Bubnov–Galerkin method for each element and the conditioning procedure described by Mitchell and Wait [6], we obtain the system of partial differential equations

$$[A]\frac{\partial\{\Theta\}}{\partial \mathrm{Fo}} + [B]\frac{\partial\{\Theta\}}{\partial Z} = [C]\frac{\partial^2\{\Theta\}}{\partial Z^2} + [D]\{\Theta\} + \{F\}. \tag{6}$$

We solve the system (6) by the FDM, using the scheme of Saul'ev [7].

The results of calculating the temperature at the nodal points of the elements have been used as the basis for investigating the heat-transfer process for $\mathrm{Pe} = 25$, $\mathrm{Bi} = \frac{\alpha_1 a}{\lambda_1} = 50$,

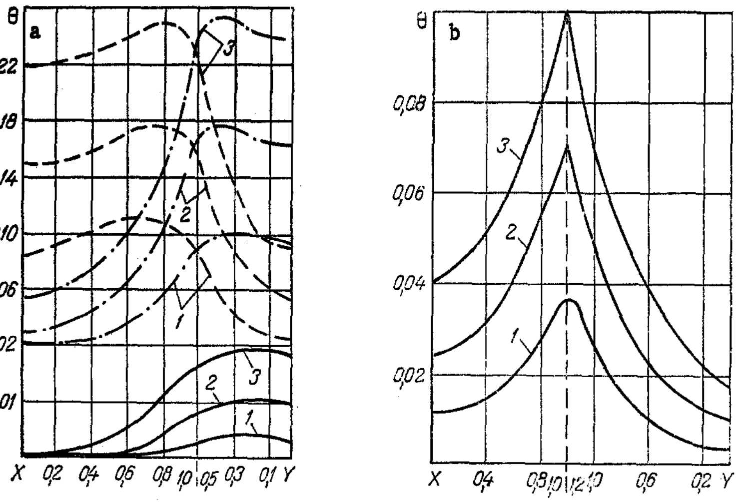

Fig. 2. Temperature distribution at the fluid–wall boundary for Fo = 0.08. a) γ = 0.5; b) 1.2; 1) Z = 0.1; 2) 0.2; 3) 0.4. $k_\alpha = 0.02$; $k_\lambda = 0.025$ and for various configurations of the duct cross section: a) $\gamma_1 = \gamma_2$; b) $\gamma_1 \neq \gamma_2$ ($k_\lambda = \lambda_2/\lambda_1$).

The objective of the numerical experiments was to study the influence of the configuration of the outer surface $S_1$ of the wall on the temperature field in the duct.

Figure 2a shows the distribution curves of the dimensionless temperature at the fluid–wall boundary ($S_2 \cup S_3$) in the case where $S_1$ is described by the equation for an ellipse ($\gamma_2 : \gamma_1 = 0.4$; $\beta = \gamma_2 - \gamma = 0.3$; dashed curves) and the equation for a circle ($\gamma_1 = \gamma_2 = 2$; $\beta = 1.5$; solid curves). Cauchy boundary conditions are specified on the outer surface of the duct wall.

It is evident from the figure that if $\gamma_1 \neq \gamma_2$, the values of the temperature are tenfold greater on the average at the boundary $S_2$, because the wall thickness in the Y direction is smaller than for $\gamma_1 = \gamma_2$. A smoother temperature variation is observed when the rate of change of the wall thickness along the coordinate axes is insignificant.

We have also investigated the temperature distributions in the duct when the equation for the outer surface of the wall is left unchanged, but the other geometrical characteristics of the duct are varied. If the wall thickness β is decreased (increased) for $\gamma_1 = \gamma_2$, the temperature at the fluid–wall boundary will increase (decrease). For example, reducing β by one half causes the temperature to increase on the average 30 fold at the boundary $S_2$ and 12-fold at $S_3$ (Fig. 2a: $\gamma_1 = \gamma_2 = 1.25$; $\beta = 0.75$; dot–dash curves).

A variation of the temperature at the fluid–wall boundary also takes place when the ratio γ between the sides of the rectangle is varied. Figure 2b shows the distribution of the dimensionless temperature at the fluid–wall boundary for a 2.4-fold increase in γ ($\gamma_1 = \gamma_2 = 2$; $\beta = 0.8$).

It is evident from the figure that the temperature increases on the average 21-fold at the boundary $S_2$ and twofold at $S_3$.

These results show that the temperature fields in the wall and in the fluid flow are interrelated and the configuration of the duct cross section has a strong influence on the temperature distribution, further corroborating the need to investigate the heat-transfer process in ducts in the conjugate formulation of the problem.

On the basis of the projection-net method we have developed a package of programs that can be used to investigate heat transfer in ducts of practically any cross-sectional configuration, to analyze the influence of time dependence, the geometry and properties of the material of the wall, and other parameters on the heat-transfer process, and to solve problems with allowance for energy dissipation in the flow, in the presence of heat sources, etc.

In addition, the program package can be used for heat-transfer computations aimed at determining the duct configuration in order to impart predetermined characteristics to the process, and in the first approximation it can serve as a "heat designer" in the design of heat-exchange equipment.

## NOTATION

$\theta_1$, $\theta_2$, X, Y, Z, Fo, dimensionless variables: temperatures, coordinates, and time; $a$, b, sides of rectangle; $a_1$, $a_2$, $\lambda_1$, $\lambda_2$, thermal diffusivities and thermal conductivities of wall material and fluid; $\gamma_1$, $\gamma_2$, dimensionless semiaxes of ellipse; $\beta$, thickness of duct wall in Y direction; Pe, Bi, Péclet and Biot numbers; $W_z(X, Y)$ dimensionless velocity profile of fluid flow in duct.

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# FUNDAMENTAL ASPECTS OF THE DEVELOPMENT OF ALGORITHMS FOR MATHEMATICAL MODELING OF THE THERMAL MODE OF THIN-WALLED STRUCTURES


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Specialized algorithms are proposed for computation of the temperature fields in thin-walled structural elements.


Among the universal methods of mathematical modeling of the thermal mode of a structure should be those based on solving systems of heat-conduction equations [1, 2]. Application of the method of "skeleton" structures [2] permits computing the temperature fields in structures of practically any geometry. Its universality lies in the fact that the "skeleton" structure combines the thermal models of the individual elements into a single generalized mathematical model. Moreover, it can also be used to compute the temperature fields in elements of complex geometry. For this, the element is partitioned into separate subdomains of canonical shape whose thermal state is described by the traditional heat-conduction equations. However, such a breaking down of the structural elements results in excessive awkwardness of the mathematical model and degrades its graphic appearance and convenience of application. Hence, the construction of typical methods and recipes for the solution of problems of analyzing temperature fields in groups of structural elements or individual elements of complex shape possessing definite characteristic criteria which would permit expansion of the domain of application of the method of "skeleton" structures is urgent. This paper is devoted to the development of algorithms to solve this problem.

The paper [3], in which an algorithm is proposed for the computation of temperature fields in thin-walled structural elements having the longitudinal coordinate z common for all plates, might be an example of the development of specialized algorithms.

Let us first examine the problem of computing the thermal state of the plates displayed in Fig. 1a. The temperature distribution in these plates is described by using the following system of heat-conduction equations


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